

# SYLLABUS OF MATHEMATICS

## UGMM-01 Calculus

### Block-I Real Numbers and Functions

Basic properties of  $\mathbf{R}$ , Absolute value, Intervals on the real line, Functions (Definition and examples), Inverse functions, Graphs of functions, Operations on functions, Composite of functions, Even and odd functions, Monotone functions, Periodic functions.

Definition of limits, Algebra of limits, Limits as  $x \rightarrow \infty$  (or  $-\infty$ ), One-sided limits, Continuity (Definitions and Examples, Algebra of continuous functions)

Definition of derivative of a function, Derivatives of some simple functions, Algebra of derivatives, The chain rule, Continuity versus derivability.

Derivatives of the various trigonometric functions, Derivatives of inverse function, The inverse function theorem, Derivatives of inverse trigonometric functions, Use of transformations. Derivative of exponential function, Logarithmic functions, Hyperbolic functions, Inverse hyperbolic functions, Methods of differentiation (Derivative of  $x^f$ , Logarithmic Differentiation, Derivatives of functions defined in terms of a parameter, Derivatives of implicit functions.

### Block-II Drawing Curves

Second and third order derivatives,  $n$ th order derivatives, Leibniz theorem, Taylor's series and Maclaurin's series Maxima-minima of functions (Definitions and examples, a necessary condition for the existence of extreme points), Mean value theorems (Rolle's theorem, Lagrange's mean value theorem), Sufficient conditions for the existence of extreme points (First derivative test, Second derivative test), Concavity/convexity, Points of inflection. Equation of tangents and normals, Angles of intersection of two curves, Tangents at the origin, Classifying singular points, Asymptotes (Parallel to the axes, Oblique asymptotes). Graphing a function, Tracing a curve (given its Cartesian equation, or in parametric form, or Polar equation).

### Block-III Integral Calculus

Partitions of a closed interval, Upper and lower product sums, Upper and lower integrals, Definite integral, Fundamental theorem of calculus. Standard integrals, Algebra of integrals, Integration by substitution, Integrals using trigonometric formulas, Trigonometric and Hyperbolic substitutions, Two properties of definite integrals, Integration by parts, Evaluation of

$\int (a^2 - x^2)dx, \int (a^2 + x^2)dx, \int (x^2 - a^2)dx, \int e^x (f(x) + f'(x))dx$ . **Reduction formulas for**  
 $\int \sin^n x dx, \int \cos^n x dx, \int \tan^n x dx$  and  $\int \sec^n x dx$ , **Integrals involving products of trigonometric**

**functions (Integrand of the type  $\sin^m x \cos^n x, e^{ax} \sin^n x$ ), Integrals involving hyperbolic functions.**

Integration of some simple rational functions, Partial fraction decomposition, Method of substitution, Integration of rational trigonometric functions, Integration of Irrational functions.

### Block-IV Application of Calculus

Monotonic functions, Inequalities, Approximate value. Area under a curve (Cartesian equation, Polar equations), Area bounded by a closed curve, Numerical integration. (Trapezoidal rule, Simpson's rule). Length of a plane curve (Cartesian form, Parametric form, Polar form), Volume of a solid of revolution, Area of surface of revolution.

## UGMM-02

### Linear Algebra

#### Block-I Vector Space

Sets, subsets, union and intersection of sets, Venn diagrams, Cartesian product, relations, functions, composition of functions, binary operations, fields. Plane and space vectors, addition and scalar multiplication of vectors, scalar product, orthonormal basis, vector equations of a line, plane and sphere. Definition and basic properties, subspaces, linear combination, algebra of subspaces, quotient spaces. Linear independence and some results about it, basic results about basis and dimension, completion of a linearly independent set to a basis, dimension of subspaces and quotient spaces.

#### Block-II Linear Transformation and Matrices

Definitions and examples of linear transformation, kernel, range space, rank and nullity, homomorphism theorems.  $L(U, V)$ , the dual space, composition of transformations, the minimal polynomial. Definition of a matrix, matrix associated to a linear transformation, the vector space  $M_{m \times n}(F)$ , transpose, conjugate, diagonal and triangular matrices, matrix multiplication, inverse of a matrix, matrix of a change of basis. Rank of a matrix, elementary operations, row-reduced echelon matrices, applying row reduction to obtain the inverse of a matrix and for solving a system of linear equations.

#### Block-III Eigen Values and Eigenvectors

Definition and properties, product formula, matrix adjoint and its use for obtaining inverses, Cramer's rule, determinant rank. Definition and how to obtain them, diagonalisation. Cayley-Hamilton theorem, minimal polynomial's properties. Definition, norm of a vector, orthogonality.

#### Block-IV Inner Product and Quadratic Forms

Linear functionals of inner product spaces, adjoint of an operator, self-adjoint and unitary operators, Hermitian and unitary matrices. Definitions, representation as matrix product, transformation under change of basis, rank of a form, orthogonal and normal canonical reductions. Definitions, standard equations, description and some geometrical properties of an ellipse, a hyperbola and parabola, the general reduction.

## UGMM-04

### Elementary Algebra

#### Block-I Solutions of Polynomial Equations

Definition and examples of sets and subsets, Venn diagrams, Complement, Intersection, Union, Distributive laws, De Morgan's laws, Cartesian product. What a complex number is, Geometrical representation, Algebraic operations, De Moivre's theorem, Trigonometric identities, Roots of a complex number. Recall of solutions of linear & quadratic equations, Cubic equations (Cardano's solution, Roots and their relation with coefficients), Biquadratic equations (Ferrari's solution, Descartes' solution, Roots and their relation with coefficients).

**APPENDIX:** Some mathematical symbols (Implication, two-way implication, for all, there exists), Some methods of proof (Direct proof, contrapositive proof, proof by contradiction, proof by counter-example).

Linear systems, Solving by substitution, Solving by elimination. Definition of a matrix, Determinants, Cramer's rule. Inequalities known to the ancients (Inequality of the means, Triangle inequality), Less ancient inequalities (Cauchy-Schwarz inequality, Weierstrass' inequalities, Tchebyshev's inequalities).

## **UGMM-05**

### **Analytical Geometry**

#### **Block-I Conics**

Equations of a line, Symmetry, Change of axes (Translating the axes, rotating the axes), Polar coordinates. Focus-directrix property, Description of standard form of parabola, ellipse and hyperbola; Tangents and normals of parabola, ellipse, hyperbola; Polar equation of conics. General second degree equation, Central and non-central conics, tracing a conic (Central conics, Parabola), Tangents, Intersection of conics.

#### **Block-II Sphere, Cone and Cylinder**

Points, Lines (Direction cosines, Equations of a straight line, Angle between two lines), Planes (Equations of a plane, Intersecting planes and lines). Equations of a sphere, Tangent lines and planes, Two intersecting spheres, Spheres through a given circle Cones, Tangent plane to a cone, Cylinders.

#### **Block-III Conicoids**

Definition of a conicoid, Change of axes (Translation of axes, projection, Rotation of Axes), Reduction to standard form. A conicoid's centre, Classification of central conicoids, Ellipsoid, Hyperboloid of one sheet, Hyperboloid of two sheets, Intersection with a line or a plane. Standard equation, Tracing the paraboloids, Intersection with a line or a plane

## **UGMM-06**

### **Abstract Algebra**

#### **Block-I Elementary Group Theory**

Sets, Cartesian Product, Relations, Functions, Some number theory – Principle of induction and divisibility in  $\mathbb{Z}$ . Binary operations, Definition of a group, Properties of a Group, Some details of  $\mathbb{Z}_n$ ,  $S_n$ ,  $\mathbb{C}$ , and appendix on some properties of complex numbers. Subgroups and their properties, Cyclic groups. Cosets; Statement, proof and applications of Lagrange's theorem

#### **Block-II Some More Group Theory**

Definition and standard properties of normal subgroups, Quotient groups. Definition and examples, Isomorphisms, Isomorphism theorems, Automorphisms. Definition, Examples, Cayley's theorem. Direct product, Sylow theorems (without proof), Classifying groups of order 1 to 10.

#### **Block-III Elementary ring Theory**

Elementary properties, Examples of commutative and non-commutative rings and rings with and without identity. Definitions, Examples, Standard properties, Quotient Rings (in the context of commutative rings).

#### **Block-IV Integral Domains and Fields**

Definition and properties of integral domains, Fields, Prime and maximal ideals, Fields of quotients. Examples, Division Algorithm and Roots of Polynomials. Euclidean domain, PID, UFD. Eisenstein's criterion, Prime fields, Finite fields.

## UGMM-08

### Differential Equations

#### Block-I Ordinary Differential Equations of First Order

Basic concepts in the theory of differential equations, Family of curves and differential equations, Differential Equations arising from physical situations. Separation of Variables, Homogeneous equations, Exact equations, Integrating factors. Classification of first order differential equations (DE), General solutions of linear non-homogeneous equation, Method of Undetermined coefficient, Method of Variation of Parameters, Equations reducible to linear form, Applications of linear DEs. Equations which can be factorized, Equations which cannot be factorized (Solvable for x, y, independent variable absent, homogeneous in x and y, Clairaut's and Riccati's equations.

#### Block-II Second and Higher Order Ordinary

General form of linear ordinary differential equation, Condition for the existence of unique solution, linear dependence and independence of the solution of DEs, Method of solving homogeneous equation with constant coefficients. Types of non-homogeneous terms for which the method is applicable (polynomial, exponential, sinusoidal etc.), Observations and Constraints of the method. Variation of parameters, Reduction of order, Euler's equations. Differential operators, General method of finding Particular Integral (PI), Short method of finding PI, Euler's equations. Method of changing independent Variable, Method of changing dependent variable, Applications – Mechanical Vibrations, Electric Circuits

#### Block-III First Order Partial Differential Equations

Frames of reference for curves and surfaces, Basic concepts in 2-dimensions, Curves and surfaces in space

Formation of simultaneous DEs, Existence and Uniqueness, Methods of solution of

$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ , Applications – Orthogonal trajectories, Particle motion in phase-space, Electric Circuits.

Formation of Pfaffian Differential Equations their geometrical meaning, Integrability, Methods of Integration (Variable separable, One variable separable, Homogeneous PfDEs, Natani's method). Origin, Classification and Solution of linear first order PDEs, Linear Equations of the First Order, Cauchy Problem. Complete integral, Compatibility, Charpits method, Standard forms, Jacobi's method, Cauchy problem.

#### Block-IV Second and Higher Order Partial Differential Equations

General form of partial differential equation of any order, – Classification and the Integral, Solutions of reducible homogeneous equations, Solutions of irreducible homogeneous equations. Particular integral, Analogies of Euler's equations Origin of second order PDEs, Classification, Variable separable solution for Heat flow, Wave and Laplace equations

## UGMM-09

### Real Analysis

#### Block-I Real numbers and Functions

Sets and functions, system of real numbers, mathematical induction

Order relations in real numbers, algebraic structure (ordered field, complete ordered field), countability. Neighbourhood of a point, open sets, limit point of a set (Bolzano-Weierstrass Theorem), closed sets, compact sets (Heine-Borel Theorem, without proof)

Algebraic functions, transcendental functions, some special functions.

### **Block-II Sequences and Series**

Sequences, bounded sequences, monotonic sequences, convergent sequences, criteria for the convergence of sequences, Cauchy sequences, algebra of convergent sequences.

Infinite series, general tests of convergence, some special tests of convergence (D'Alembert's ratio test, Cauchy's integral test, Raabe's test, Gauss's test)

Alternating series (Leibniz's test), absolute and conditional convergence, rearrangement of series.

### **Block-III Limit and Continuity**

Notion of limit (finite limits, infinite limits, sequential limits), algebra of limits

Continuous functions, algebra of continuous functions, non-continuous functions

Continuity on bounded closed intervals, pointwise continuity and uniform continuity.

Derivative of a function (geometrical interpretation), differentiability and continuity, algebra of derivatives, sign of a derivatives. Rolle's theorem, mean value theorems (Lagrange, Cauchy and generalised mean value theorems), intermediate value theorem for derivatives (Darboux theorems).

Taylor's theorem, Maclaurin's expansion, indeterminate forms, extreme values

### **Block-IV Differentiability**

Riemann integrability, Riemann integrable functions, Algebra of integrable functions, computing an integral. Properties of Riemann integral, Fundamental Theorem of Calculus, mean value theorems.

Sequences of functions, Pointwise convergence, uniform convergence (Cauchy's criterion), series of functions.

## **UGMM-10**

### **Numerical Analysis**

#### **Block-I Solutions of Non-Linear Equations in one Variable**

Review of Calculus- Intermediate Value Theorem Rolle's Theorem Lagrange's Mean Value Theorem, Taylor's Theorem, Round off Error Truncation Error; Iteration Methods for Finding Roots- Initial Approximation to a Root, Bisection Method, Fixed Point Iteration Method Chord Methods for Finding Roots- Regula Falsi Method, Newton Raphson Method Convergence Criterion; Approximate Roots of Polynomial Equations- Birge Vieta Method Graeffe's Root Squaring Method

#### **Block-II Solutions of Linear Algebraic Equations**

Direct Methods- Preliminaries Cramer's Rule, Gauss, Elimination Method, LU Decomposition Method; Inverse of Square Matrix- Method of Adjoints, The Gauss- Jordan Reduction Method, LU Decomposition Method; Iterative Method- General Iteration Method, Jacobi Iteration Method, The Gauss- Seidel Iteration Method ;

Eigen Values and Eigen Vectors- The Eigen Value Problem, the Power Method, The Inverse power method

#### **Block-IV Interpolation**

Lagrange's Form; Newton Form of the Interpolating Polynomial; Interpolation at Equally Spaced Points- Forward and Backward Differences, Newton's Forward and Backward

Difference Formula

**Block-V Numerical Differentiation, Integration and Solutions of Differentiation Equations**

Numerical Differentiation ; Numerical Integration; Numerical Solutions of Ordinary Differential Equations- Taylor Series Method, Euler's Method, Richardson's Extrapolation  
Numerical Solutions of Numerical Equations Using Runge- Kutta Methods- 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> Order.

**UGMM-11**

**Probability and Statistics**

**Block-I Descriptive Statistics**

Frequency Distribution of a Character; Measure of Central Tendency and Dispersion; Skewness and Kurtosis; Correlation and Regression

**Block-02 Probability on Discrete Sample Spaces**

Sample Space of a Random Experiment ; Probability on a Discrete Sample; Discrete Random Variable and its Probability Distribution- Random Variable, Joint and Marginal Distribution and its independence, Mathematical Expectation, Moments and Moments Generating Functions, Covariance; Standard Probability Distribution-I- Bernoulli Distribution, Binomial Distribution, Hyper geometric Distribution ;  
Standard Probability Distribution-II- Geometric Distribution, Negative Binomial Distribution, Poisson Distribution.

**Block-III Distribution Theory**

Univariable Distributions- Distribution Functions, Density Functions, Expectation and Variance, Moments and Moments Generating Functions.; Standard Continuous Distributions- Normal Distribution, Exponential and Gamma Distribution, Beta Distribution.; Bivariate Distribution Density Functions, Distribution Functions, Conditional Distribution, Independence Expectations Correlation and Regression.; Functions Random Variables- Direct Approach Transformation Approach, Chi-Square Distribution, Independence Expectations, Correlation and Regression  
Limit Theorems- Chebyshev's Inequality, Weak Law of Large Numbers, Poisson Approximation to Binomial, Central Limit Theorem.

**Block-IV Elements of Statistical Inference**

General Introduction- Inductive Inference, Random Sampling, Sampling Distributions, Related to Normal Distribution, Point Estimation Testing of Hypothesis, Interval Estimation  
Point Estimation- Properties of Estimators, Method of Moments, Method of Maximum Likelihood.

Testing of Hypotheses- Some Concepts, Neyman- Pearson Lemma, Likelihood- Ratio Tests  
Common Tests and Confidence Intervals- Some Common Tests of Hypothesis for Normal Populations, Confidence Intervals, Chi- Square test for Goodness of Fit.

**UGMM-12**

**Linear Programming**

**Block-I Basic Mathematics and Optimization**

Basic Algebra- Matrices and Determinants, Vector; Inequalities and Convex Sets; Optimization in two Variables; Optimization in More Than Two Variables

**Block-II      Simplex Method and Duality**

Standard Form and Solutions ;Simplex Method; Primal and Dual; Duality Theorems

**Block-III      Special Linear Programming Problems**

Transportation Problem ; Feasible Solution of the Transportation; The Assignment Problem

**Block-IV      Game Theory**

Games With Pure Strategies; Games With Mixed Strategies; Graphical Method and Dominance;  
Games and Linear Programming

